Some Equilibrium Properties of a Rate Control Protocol

Abhijit Kiran Valluri

IIT Madras

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Outline

Introduction

- Current day implementation of Transport Control Protocols (TCP) are unable to provide the high quality of service that users expect from the Internet.
  - Studies have shown that Additive Increase Multiplicative Decrease (AIMD) TCP is unsuitable for next generation networks.
- Explicit congestion control protocols promise to provide a fair and stable network.

Problem statement

We consider two different models of RCP that have been suggested in literature.

- The first model considered explicitly models the rate dynamics and the queue size dynamics with a switch.
  - The non-switched dynamics better represents the packet level operation of the protocol.
  - Necessary and sufficient conditions for the local stability of the model were developed.
  - The nonlinear effects the occur on violating the stability conditions were studied using computations.
- The second model models the queue size dynamics, in the rate evolution equation, as a deterministic representation of the underlying stochastics.
  - Local stability and local bifurcation analysis has been performed.
  - Further properties of RCP with regard to multiple time scales, utilisation, queue feedback, etc. have also been studied.

Relevance of this work

Explicit congestion control protocols have a claim to be able to provide a network with:

- high and controllable utilisation,
- very low loss,
- high throughput,
- short flow completion times.

As this is an engineered system, we can control the microscopic rules to give the desirable macroscopic properties.

Model A

The following equations depict the Model A:

$$\frac{d}{dt}R(t) = \frac{R(t)}{C\overline{\overline{T}}}\left(a(C - y(t)) - \beta\frac{q(t)}{\overline{\overline{T}}}\right) \tag{1}$$

where

$$y(t) = \sum_s R(t - T_s) \tag{2}$$

and

$$\begin{aligned} \frac{d}{dt}q(t) &= [y(t) - C] \qquad & q(t) > 0 \\ &= [y(t) - C]^+ \qquad & q(t) = 0. \end{aligned} \tag{3}$$

Model A

The following diagram represents results from some packet level simulations on Model A.

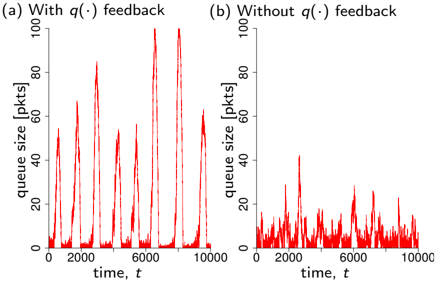

Figure: Traces from a packet-level simulation of a single bottleneck link with capacity of 1 packet per unit time, 100 RCP sources, round trip time of 100 time units, and a target link utilisation of 90%. The parameter values used are (a) $a = 0.5$; $\beta = 1$, and (b) $a = 1$; $\beta = 0$; $\gamma = 0.9$.

Model A

The key observations from this figure are

- The mean queue size is not zero. Therefore, the switch in the queue dynamics is not vital.
- The system appears to be in a nonlinear oscillations with queue feedback, while it shows a stochastic behaviour without queue feedback.

The two forms of feedback are playing a non-trivial role that is not happen in a linear system. A thorough analysis of the stability of Model A is needed.

Stability analysis

The necessary and sufficient condition for the local stability of the "non-switched" version of Model A, without queue feedback, is:

$$a < \frac{\pi}{2}.$$

The necessary and sufficient condition for the local stability of the "non-switched" version of Model A, with queue feedback, is:

$$\tan \frac{\sqrt{a^2 + \sqrt{a^4 + 4\beta^2}}}{\sqrt{2}} < \frac{a}{\beta} \frac{\sqrt{a^2 + \sqrt{a^4 + 4\beta^2}}}{\sqrt{2}}.$$

Numerical computations

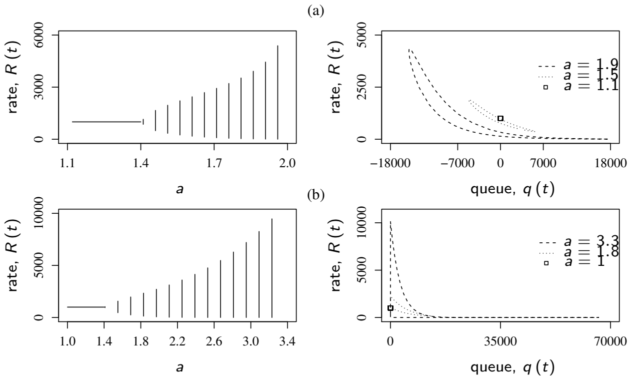

Figure: Numerical computations for Model A: (a) Bifurcation diagram (left) and phase portrait (right) for the "non-switched" case, with $\beta = 0.3$ and a is varied, (b) Bifurcation diagram (left) and phase portrait (right) for the "switched" case, with $\beta = 0.3$ and a is varied. Other parameter values are: RTT, $T = 0.1$ time units; Capacity, $C = 100,000$ packets per unit time; Number of flows, $s = 100$.

Numerical computations

- The phase portraits in the above plots provide computational evidence for the existence of limit cycles as we violate the conditions for stability.
- The plots provide evidence for the existence of a bifurcation at the edge of the stability region.
- Both the cases of switched and non-switched systems are topologically equivalent. Moreover, the packet level behaviour of the system is better represented by the non-switched system. Therefore, it is reasonable to analyse the stability of the non-switched system.

Model B

The following equations depict the Model B:

$$\frac{d}{dt}R_j(t) = \frac{aR_j(t)}{C_j\overline{T}_j(t)}\left(C_j - y_j(t) - b_jC_jp_j(y_j(t))\right) \tag{4}$$

where

$$y_j(t) = \sum_{r:j\in r} x_r(t - T_{rj}) \tag{5}$$

and

$$\overline{T}_j(t) = \frac{\sum_{r:j\in r} x_r(t)\, T_r}{\sum_{r:j\in r} x_r(t)} \tag{6}$$

with,

$$x_r(t) = w_r\left(\sum_{j\in r} R_j(t - T_{jr})^{-1}\right)^{-1}, \tag{7}$$

and

$$p_j(y_j) = \frac{y_j\sigma_j^2}{2(C_j - y_j)}. \tag{8}$$

Stability analysis

The necessary and sufficient condition for the local stability of Model B, with queue feedback, is

$$a\left(2+\frac{b}{4}-\sqrt{\frac{b^2}{16}+\frac{b}{2}}\right)<\frac{\pi}{2}.$$

The sufficient condition for the local stability of Model B, with queue feedback, is

$$a<\frac{\pi}{4}.$$

The necessary and sufficient condition for the local stability of Model B, without queue feedback, is

$$a<\frac{\pi}{2}.$$

Stability analysis

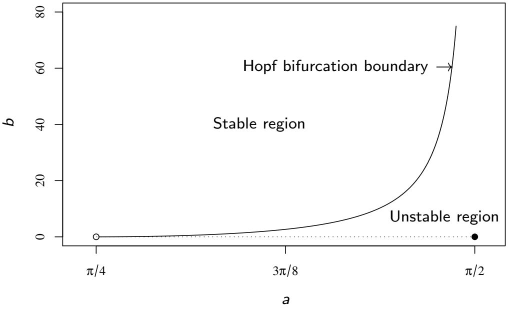

Figure: Stability chart for Model B

The boundary of the stability region is the Hopf bifurcation boundary as it is shown that it satisfies the transversality condition for Hopf bifurcation.

Further properties of RCP

We consider the impact of

- multiple time scales,
- queue feedback,
- utilisation, and
- parameter a

on the properties of RCP, with respect the Model B.

Multiple time scales

There exist four key time scales, which impact the dynamics of RCP, in the following ranges relative to queuing delay (abbreviated as qd): $\gg qd$, $> qd$, $\approx qd$ and $< qd$.

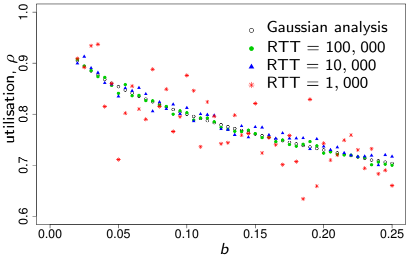

Figure: Utilisation, ρ, measured over one RTT, for different values of the parameter b with 100 RCP sources sending Poisson traffic.

Multiple time scales

The salient features of the above figure are:

- The curve agrees very well with the theoretical, Gaussian analysis curve for high RTT of 100,000 time units.
- As RTT is reduced, there is an increased variability in the utilisation.
- Hence, RTT plays a significant in the stability and the dynamics of the system, which is not predicted by our linear stability analysis.

We further observe that RTT also impacts the bifurcation occurring at the edge of the stability region; see the thesis for the corresponding figure.

Queue feedback

In section 2, we have shown that the queue size dynamics presents a limit cycle with queue feedback, while it is stable without queue feedback.

Further numerical computations performed on Model B, presented in the thesis, suggest that the limit cycles obtained are stable.

We also note, from the phase portraits obtained (see thesis for the plots), that both the cases, namely with and without queue feedback, are topologically equivalent.

Utilisation and parameter a

A key observation is that as utilisation decreases, the stability of the system increases, as can be noted from the bifurcation diagrams presented in the thesis.

Parameter a affects:

- stability of the system and bifurcation occurring at the edge of the stability region, as can be ascertained from the stability criteria obtained in this thesis;
- rate of convergence of the system (analysis is shown in the thesis).

We note that at $a = 1/e$, the rate of convergence of the system is maximum. Hence, the optimal value of a appears to be $1/e$.

Contributions

Our key contributions in this thesis are as follows:

- We derived the necessary and sufficient conditions for local stability for the two models of RCP considered, under certain conditions.
- We studied the consequences of violating these stability conditions in both the models.
  - In particular, we showed the existence of a Hopf type bifurcation, computationally for one and analytically for the other model.
- We highlighted the potential destabilising effect of having two forms of feedback.

We believe that this sheds light on the issue of implementing queue feedback in the protocol. Further, the occurrence of Hopf type bifurcation due to the queue feedback raises further questions regarding the impact of the nonlinear properties of the fluid model on the protocol design.

Future work

We shall now outline avenues for future work.

- Analytically derive that both models exhibit a Hopf type bifurcation at the edge of the stability region.
- Verify, analytically, the stability of the emerging limit cycles as the stability conditions are violated.
- Ascertain the impact of the parameters on the stochastic effects in the small buffer variant of RCP (i.e. Model B).

This would aid us in obtaining a deeper understanding of both the nonlinear and the stochastic effects of the protocol and role of queue feedback.

Questions?